MTH 167 Precalculus with Trigonometry Formula Sheet

Distance Formula

The distance d between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula

The midpoint M of the line segment connecting the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is:

$$M = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2} \right)$$

Quadratic Formula: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Point-Slope form of a line: $$y - y_1 = m(x - x_1)$$

Parallel lines have the same slope: $m_1 = m_2$
Perpendicular lines have negative reciprocal slopes: $m_1 = -\dfrac{1}{m_2}$

Exponential and Logarithmic Formulas

Interest equations: $A = Pe^{rt}$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Growth and Decay: $A = A_0 e^{kt}$

Change of Base $\log_a x = \dfrac{\log_b x}{\log_b a}$

Conic Sections

Standard Equations for Parabolas

Horizontal axis (opens left/right)	Vertical axis (opens up/down)
$(y-k)^2 = 4p(x-h)$ If $p > 0$ it opens right If $p < 0$ it opens left	$(x-h)^2 = 4p(y-k)$ If $p > 0$ it opens up If $p < 0$ it opens down

Vertex always located at (h, k).

Focus: p units from the vertex on the axis of symetry.

Directrix: p units from the vertex.

Standard Equations for Ellipse

Center always located at (h, k).

$$\left.\begin{array}{l}\text{Major axis length: } 2a \\ \text{Minor axis length: } 2b\end{array}\right\} 0 < b < a.$$

Horizontal major axis.	Vertical major axis.
$\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$	$\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$

The foci lie on the major axis c units from the center with

$$c^2 = a^2 - b^2.$$

Standard Equations for Hyperbola

Center always located at (h, k).

$$\left.\begin{array}{l}\text{Transverse axis length: } 2a \\ \text{Conjugate axis length: } 2b\end{array}\right\} a > 0 \text{ and } b > 0$$

Horizontal transverse axis. (x-intercepts)	Vertical transverse axis. (y-intercepts)
$\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$	$\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

The foci lie on the transverse axis c units from the center with

$$c^2 = a^2 + b^2.$$

The asymptote "box" for drawing is determined by

$\pm a$ along the transverse axis from the center

$\pm b$ along the conjugate axis from the center

Trigonometric Formulas

Sum and Difference Formulas

$$\sin(u+v) = \sin(u)\cos(v) + \cos(u)\sin(v) \qquad \cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$$

$$\sin(u-v) = \sin(u)\cos(v) - \cos(u)\sin(v) \qquad \cos(u-v) = \cos(u)\cos(v) + \sin(u)\sin(v)$$

$$\tan(u+v) = \frac{\tan(u)+\tan(v)}{1-\tan(u)\tan(v)} \qquad \tan(u-v) = \frac{\tan(u)-\tan(v)}{1+\tan(u)\tan(v)}$$

Double Angle Formulas

$$\sin(2u) = 2\sin(u)\cos(u)$$

$$\tan(2u) = \frac{2\tan(u)}{1-\tan^2(u)}$$

$$\begin{aligned} \cos(2u) &= \cos^2(u) - \sin^2(u) \\ &= 2\cos^2(u) - 1 \\ &= 1 - 2\sin^2(u) \end{aligned}$$

Half Angle Formulas

$$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos(u)}{2}} \qquad \cos\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1+\cos(u)}{2}}$$

$$\tan\left(\frac{u}{2}\right) = \frac{1-\cos(u)}{\sin(u)} = \frac{\sin(u)}{1+\cos(u)}$$

Power Reducing Formulas

$$\sin^2 u = \frac{1-\cos 2u}{2} \qquad \cos^2 u = \frac{1+\cos 2u}{2}$$

Polar Coordinate Conversions

$$\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ x^2 + y^2 &= r^2 \end{aligned}$$

Law of Sines:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$