2.2 Graphs of tan(x), cot(x), csc(x) and sec(x)

Tangent and Cotangent Graphs

The graph of the tangent can be constructed by plotting points from Table 2.1 or by using the identity $\tan x = \dfrac{\sin x}{\cos x}$. On the graph of the tangent notice that there are vertical asymptotes at multiples of $\dfrac{\pi}{2}$. This is because $\tan x = \dfrac{\sin x}{\cos x}$ and everywhere cosine is zero tangent is undefined. You can see from the cosine graph that it has zeros at $x = \dfrac{\pi}{2} + n\pi$ where $n \in \mathbb{Z}$. Also note that the period of the tangent function is π. The graph repeats every π units, it is identical between any two asymptotes.

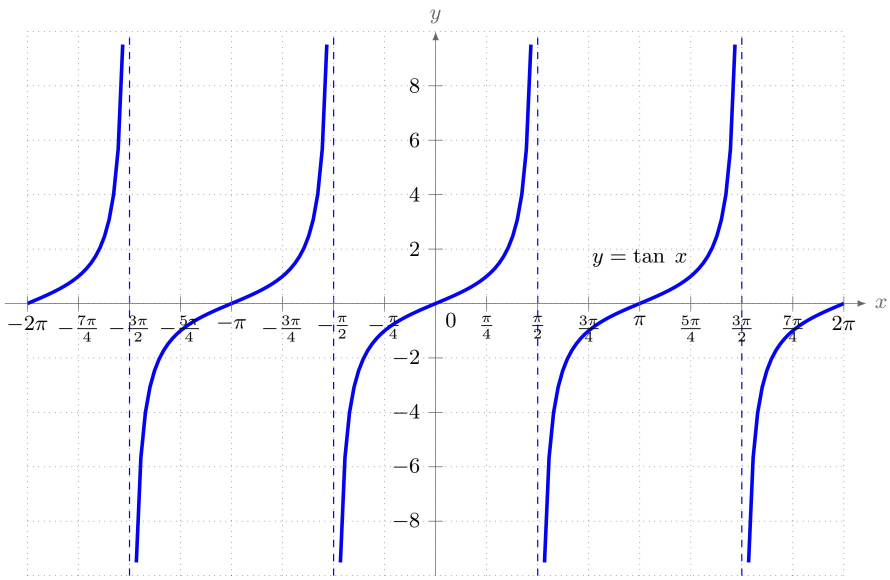

Figure 2.14: Graph of $y = \tan x$

We can perform similar transformations to what was done for the sine and cosine graphs. Those transformations are summarized here:

Summary of trigonometric transformations for tangent.

Given the function

$$y = A\tan(Bx + C) + D$$

the following transformations occur:

1. The amplitude of the function is undefined.
2. The period of the function is $\dfrac{\pi}{B}$
3. The phase shift of the function is $\dfrac{C}{B}$.
4. The vertical shift is D

A negative sign in front of the function will reflect it over the x-axis.

Example 2.2.1

Find the amplitude, period, phase shift, and vertical shift for the function $y = \frac{1}{2}\tan(2x) - 3$

Solution: The amplitude is undefined, the period is $\frac{\pi}{2}$, there is no phase shift, and the vertical shift is down 3 units.

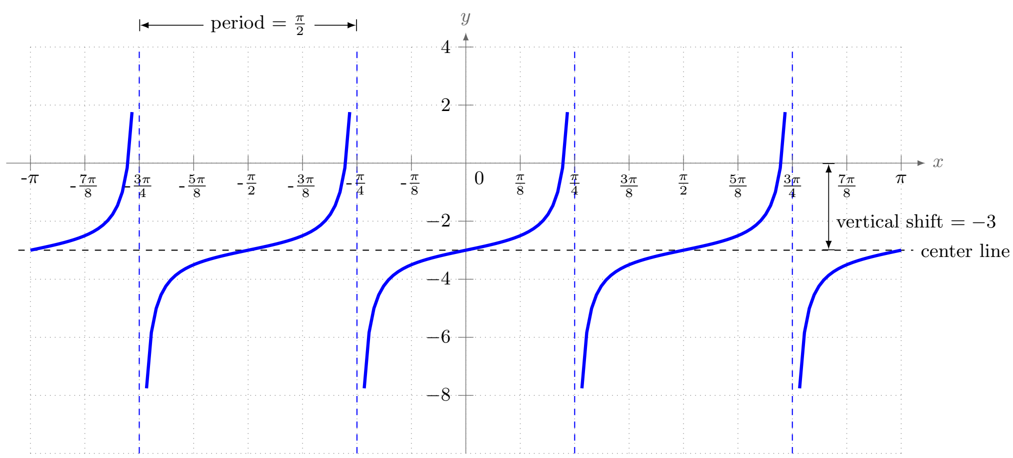

Figure 2.15: $y = \frac{1}{2}\tan(2x) - 3$

The graph of the cotangent **Figure 2.16** can be constructed by using the identity $\cot x = \dfrac{\cos x}{\sin x}$ or by using the relation $\cot x = -\tan\left(x + \frac{\pi}{2}\right)$. On the graph of the cotangent notice that there are vertical asymptotes at multiples of π. This is because $\cot x = \dfrac{\cos x}{\sin x}$ and everywhere sine is zero the cotangent is undefined. $y = \sin x$ has zeros at $x = \pi + n\pi$ where $n \in \mathbb{Z}$ so $y = \cot x$ has vertical asymptotes at $x = \pi + n\pi$. Also note that the period of the cotangent function is π. The graph repeats every π units, it is identical between any two asymptotes.

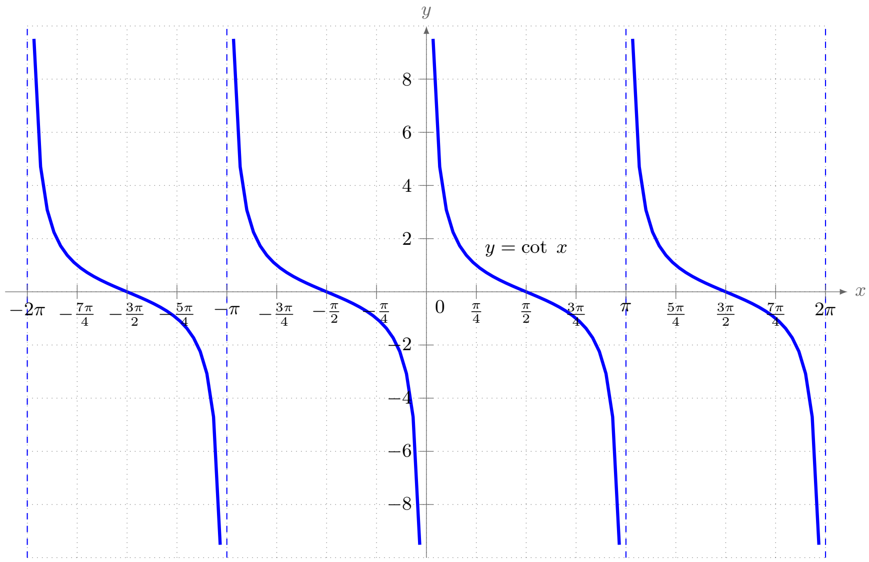

Figure 2.16: Graph of $y = \cot x$

Cosecant and Secant Graphs

The graph of the cosecant can be constructed by using the identity $\csc x = \dfrac{1}{\sin x}$. On the graph of the cosecant notice that there are vertical asymptotes at multiples of π. This is because $\csc x = \dfrac{1}{\sin x}$ and everywhere sine is zero the cosecant is undefined. The period of the cosecant function is 2π which is the same as the sine function. The graph repeats every 2π units. **Figure 2.17** shows the graph of $y = \csc x$, with the graph of $y = \sin x$ (the dashed curve) for reference.

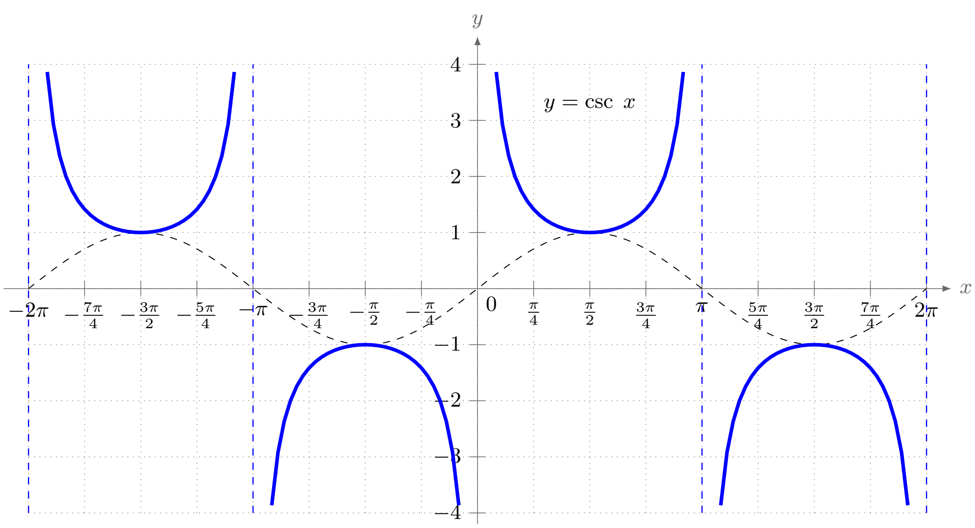

Figure 2.17: Graph of $y = \csc x$ in blue and $y = \sin x$ (dashed line)

The graph of the secant can be constructed by using the identity $\sec x = \dfrac{1}{\cos x}$. On the graph of the secant notice that there are vertical asymptotes at multiples of $\dfrac{\pi}{2}$ because the graph of $y = cosx$ has zeros at $x = \dfrac{\pi}{2} + n\pi$ where $n \in \mathbb{Z}$. The period of the secant function is 2π which is the same as the cosine function. The graph repeats every 2π units. **Figure 2.18** shows the graph of $y = \sec x$, with the graph of $y = \cos x$ (the dashed curve) for reference.

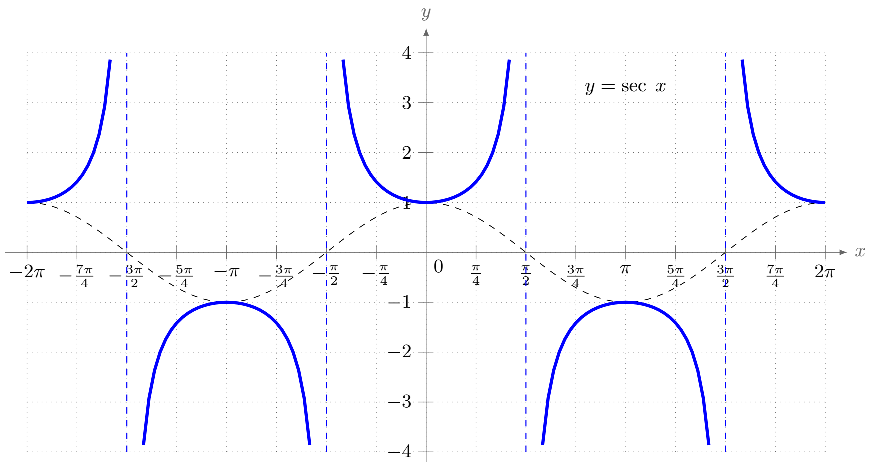

Figure 2.18: Graph of $y = \sec x$

All the same transformations that were done to the sine, cosine and tangent can be done to the other functions.

Summary of trigonometric transformations for cosecant, secant and cotangent

$y = A\csc(Bx + C)$ has undefined amplitude, period $\dfrac{2\pi}{B}$ and phase shift $\dfrac{C}{B}$

$y = A\sec(Bx + C)$ has undefined amplitude, period $\dfrac{2\pi}{B}$ and phase shift $\dfrac{C}{B}$

$y = A\cot(Bx + C)$ has undefined amplitude, period $\dfrac{\pi}{B}$ and phase shift $\dfrac{C}{B}$

A negative sign in front of the function will reflect it over the x-axis.

2.2 Exercises

For Exercises 1-9, determine the amplitude, period, vertical shift, horizontal shift, and draw the graph of the given function for two complete periods.

1. $y = 3\tan x$

2. $f(x) = -3\csc x$

3. $y = -3\sec(2x)$

4. $f(x) = -3\sec(\pi x)$

5. $y = \dfrac{\cot x}{4}$

6. $y = \cot\left(\frac{x}{4}\right)$

7. $y = \tan\left(x + \frac{\pi}{4}\right)$

8. $y = \frac{1}{2}\cot\left(x - \frac{\pi}{4}\right)$

9. $y = \sec(t) + 2$